

**2021****Time - 3 hours****Full Marks - 60**

Answer **all groups** as per instructions.  
Figures in the right hand margin indicate marks.  
Candidates are required to answer  
in their own words as far as practicable.

**GROUP - A**

1. Answer all questions and fill in blanks as required. [1 × 8]

(a)  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = \underline{\hspace{2cm}}$ .

(b) If  $y_1(x)$  and  $y_2(x)$  are linearly dependent, then what should be value of their Wronskian  $w(y_1, y_2)$  ?

(c) Name the type of differential equation  $\frac{dy}{dx} = \frac{f(x, y)}{\phi(x, y)}$ .

(d) Find value of  $\hat{i} \times (\hat{j} \times \hat{k})$ .

(e) Find the order of differential equation  $\frac{d^2y}{dt^2} = k^2 \frac{dy}{dt}$ .

(f) If position vector of a point 'P' is  $\mathbf{r} = r(u, v, w)$  then scale factors  $h_1 = \underline{\hspace{2cm}}$ ,  $h_2 = \underline{\hspace{2cm}}$  and  $h_3 = \underline{\hspace{2cm}}$ .

P.T.O.

(g)  $\delta[(x-a)(x-b)] = \underline{\hspace{2cm}}$ .

(h) If  $\nabla \times \mathbf{A} = 0$ , then what is the one value of  $\oint \mathbf{A} \cdot \hat{n} \, ds$  ?

**GROUP - B**

2. Answer any eight of the following questions within two to three sentences each. [1½ × 8

(a) State the condition for differential equation  
 $M(x, y) \, dx + N(x, y) \, dy = 0$  to be exact.

(b) Write down Taylor series of  $f(x)$  about the point  $x = a$ .

(c) What do you mean by flux of a vector ?

(d) Find  $\text{div } \mathbf{r}$ .

(e) Find Integrating factor of  $y' + 5y = 3e^x$ .

(f) What are vector and scalar fields ?

(g) Find the value of  $\frac{\partial^2 u}{\partial x \partial y}$  if  $u = x^2 + y^2$ .

(h) Write Dirac delta function in integral form.

(i) Show that  $\nabla \times \mathbf{r} = 0$ .

(j) Complete the vector identity  $\nabla \times \nabla \times \mathbf{A} = \underline{\hspace{2cm}}$ .

GROUP – C

3. Answer any eight of the following questions within 75 words each.

(a) Find value of  $m$  if  $\mathbf{A} = \hat{i} + \hat{j} + 5\hat{k}$  and  $\mathbf{B} = 2\hat{i} + m\hat{j} - \hat{k}$  are perpendicular to each other. [2 × 8]

(b) State uniqueness theorem.

(c) If the edges of a parallelepiped are  $2\hat{i} - 3\hat{j} + 4\hat{k}$ ,  $\hat{i} + 2\hat{j} - 2\hat{k}$  and  $-3\hat{i} + 2\hat{j} - \hat{k}$  then find its volume.

(d) Show that  $\delta(ax + b) = \frac{1}{|a|} \delta\left(x + \frac{b}{a}\right)$ .

(e) Find a unit vector perpendicular to surface  $x^2 + y^2 + z^2 = 7$  at the point (2, 3, 4).

(f) State Stoke's theorem.

(g) Express operator  $\nabla$  in cylindrical coordinate.

(h) Find  $\nabla\phi \cdot d\mathbf{r}$  where  $\phi(x, y, z)$  is a constant.

(i) If  $\mathbf{a}$  is a constant vector, then show that  $\text{grad}(\mathbf{r} \cdot \mathbf{a}) = \mathbf{a}$ .

(j) Using Stoke's theorem, prove

$$\int_C \mathbf{r} \cdot d\mathbf{r} = 0 \text{ where } \mathbf{r} = x\hat{i} + y\hat{j} + z\hat{k}.$$

GROUP – D

Answer **any four** questions within 500 words each.

4. (a) Find the appropriate value of  $(122)^{\frac{1}{3}}$  using differential calculus. [3]
- (b) Solve the differential equation  $x \frac{dy}{dx} + y = x^3 + x$ . [3]
5. (a) Solve the differential equation  $y'' - 5y' + 6y = 0$ . [3]
- (b) If  $y_1(x) = e^x$  and  $y_2(x) = e^{-2x}$ , show that they are linearly independent. [3]
6. Using Lagrange's method, find out the minimum value of  $x^2 + y^2 + z^2$ , when  $xyz = a^3$ . [6]
7. If  $\mathbf{A} = \hat{i} - 2\hat{j} + 3\hat{k}$ ,  $\mathbf{B} = -2\hat{i} + 3\hat{j} - 4\hat{k}$  and  $\mathbf{C} = \hat{i} - 3\hat{j} + 5\hat{k}$  then check whether they are coplanar or not. [6]
8. Prove that cylindrical coordinate system is orthogonal. [6]
9. Evaluate  $\int_{-1}^{+1} 9x^3 \delta(3x + 1) dx$ . [6]
10. Find the directional derivative of  $f = xyz$  at  $(-1, 1, 3)$  along  $\mathbf{a} = \hat{i} - 2\hat{j} + 2\hat{k}$ . [6]
11. State and prove Gauss divergence theorem. [6]

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**GROUP – A**

1. Answer all questions and fill in blanks as required. [1 × 8]
- (a) Under which condition the angular momentum of a particle be zero.
  - (b) What is the value of momentum of inertia of a sphere about its diameter when the sphere contracts to half in its radius.
  - (c) The value of Poisson's ratio should always be \_\_\_\_\_.
  - (d) Write down the Dimensional formulae of 'Kinematic Velocity'.
  - (e) For elliptical path the eccentricity should be \_\_\_\_\_.
  - (f) What is the value of 'G' in MKS system.
  - (g) Define 'stiffness'.
  - (h) What is the speed of a particle with zero rest mass ?

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**GROUP – B**

2. Answer any eight of the following questions within two to three sentences each. [1½ × 8

- (a) Write the components of angular momentum of a particle in vector form.
- (b) State 'Routh Law' for the MI of spherical bodies.
- (c) State and define 'Radius of gyration' of a body.
- (d) Define 'flexural rigidity' of a beam.
- (e) What is 'Center of Percussion' ?
- (f) Write the basic difference between a Geostationary satellite and Geosynchronous satellite.
- (g) Write the relation between time-average values of K.E. and P.E. of a body in SHM.
- (h) How is relaxation time related with resistance constant ?
- (i) Write down the postulates of special theory of relativity.
- (j) Explain the physical significance of Michelson-Morley experiment.

**GROUP – C**

3. Answer any eight of the following questions within 75 words each. [2 × 8

- (a) State and prove the law of Conservation of angular momentum.

- (b) State and Establish Parallel axis theorem of rotational bodies.
- (c) Derive an expression of K.E. in rotational motion.
- (d) A couple of 20 Nm is applied to a fly-wheel of mass 10 kg and radius of gyration 0.5 m. Find the resultant angular acceleration.
- (e) What is Coriolis force ? How its effect observe in formation of cyclone ?
- (f) Proof that a central force is a conservative force.
- (g) Proof that in an elliptical orbit, total energy depends only on the semi-major axis of the orbit.
- (h) Find the time average value of kinetic and potential energy over a period 'T' of a particle executing S.H.M.
- (i) What is quality factor (q) in damping and what will be its value in case of light damped harmonic oscillator.
- (j) Explain and deduce an expression of time dilation in relativity.

**GROUP – D**

*Answer any four questions within 500 words each.*

4. Derive an expression for the moment of inertia of a solid cylinder about its axis of symmetry. [6]

5. Deduce an expression for couple per unit twist of a uniform solid cylinder. [6]
6. Derive Poiseuille's formulae for the rate of flow of liquid through a tube. [6]
7. Deduce the differential equation of the orbit for a particle moving under central force. [6]
8. What is meant by a damped harmonic oscillator ? Write the differential equation and find its solution. [6]
9. Derive Einstein's mass-energy relation. [6]
10. Obtain a relation between Young's modulus, bulk modulus and Poisson's ratio in elasticity. [6]

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**GROUP – A**

1. Answer all questions and fill in blanks as required. [1 × 8]
- (a) What is value of 'G' in m.k.s. system ?
  - (b) Write the dimensional formula of kinematic viscosity.
  - (c) The value of Poisson's ratio lies in between \_\_\_\_\_ and \_\_\_\_\_.
  - (d) Damping tends to \_\_\_\_\_ the time period of the vibrating body.
  - (e) Does the efficiency of an engine depend on nature of working substance ?
  - (f) Is electric flux a scalar or a vector quantity ?
  - (g) Can power factor be ever equal to one ?

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- (h) Doped semiconductor is also called as \_\_\_\_\_ semi-conductor.

**GROUP – B**

2. Answer any eight of the following questions within two to three sentences each. [1½ × 8

- (a) Discuss physical significance of moment of inertia.
- (b) State Newton's law of gravitation.
- (c) Why oil spreads over water, while water does not spread over oil surface ?
- (d) How is Relaxation time related with resistance constant ?
- (e) How does velocity of sound vary with temperature ?
- (f) State the principle of superposition of waves.
- (g) Write the mathematical form of Gauss's theorem.
- (h) Under what conditions, a charged particle moving through a magnetic field experiences no force.
- (i) What do you mean by time constant of an R–C circuit.
- (j) How is N-type semiconductor produced ?

**GROUP – C**

3. Answer any eight of the following questions within 75 words each. [2 × 8

- (a) State and prove theorem of perpendicular axis.

- (b) Compare the gravitational potential at a point mid way between centre of earth and its surface with that on the surface.
- (c) If the length of a light cantilever is doubled, how will the depression of its end change ?
- (d) What do you mean by sharpness of resonance ? Under what condition is it maximum ?
- (e) How are the velocities of sound at  $t_1^{\circ}\text{C}$  and  $t_2^{\circ}\text{C}$  are related to each other ?
- (f) The efficiency of carnot cycle is  $\frac{1}{4}$ . By lowering the temperature of sink by 65 K, it increases to  $\frac{1}{2}$ . Find initial and final temperature of sink.
- (g) Explain qualitatively the oscillations of electric charge in an L-C circuit.
- (h) Distinguish between p-type and n-type extrinsic semiconductor.
- (i) A circuit has an inductance  $\frac{1}{\pi}$  H and a resistance of 2000 ohm. Find impedance offered by circuit, when 50 cycle A.C. is applied to it.
- (j) Explain Lorentz force.

GROUP – D

Answer **any four** questions within 500 words each.

4. Obtain an expression for the moment of inertia of a solid cylinder about an axis passing through its centre and perpendicular to its length. [6]
5. Derive a relation connecting Young's modulus, Bulk modulus and Poisson's ratio. [6]
6. What are Damped vibrations ? Obtain general expression for displacement of a particle vibrating in a resistive medium. [6]
7. Compare analytically, two SHMs having same periods, acting at right angles to each other and having phase difference of  $\pi$  radian. [6]
8. What is Carnot engine ? Derive an expression for the efficiency of a Carnot's engine. [6]
9. Show that the instantaneous currents during growth and decay in L-R circuit are complimentary to each other. [6]
10. Find out expressions for efficiency and ripple factor of half wave rectifier with resistive load. [6]